This thesis concentrates on a particular first-order coupled PDE system. It provides both a detailed treatment of the *existence* and *uniqueness* of monotone travelling waves to various equilibria, by differential-equation theory and by probability theory and a treatment of the corresponding hyperbolic initial-value problem, by analytic methods. Numerical techniques are also used, again both from probabilistic and analytic standpoints.

The initial-value problem is studied using characteristics to show existence and uniqueness of a bounded solution for bounded initial data (subject to certain smoothness conditions). The concept of *weak* solutions to partial differential equations is used to rigorously examine bounded initial data with jump discontinuities.

For the travelling wave problem the differential-equation treatment makes use of a shooting argument and explicit calculations of the eigenvectors of stability matrices.

The probabilistic treatment is careful in its proofs of *martingale* (as opposed to merely local-martingale) properties. A modern *change-of-measure* technique is used to obtain the best lower bound on the speed of the monotone travelling wave — with Heaviside initial conditions the solution converges to an approximate travelling wave of that speed (the solution tends to one ahead of the wave-front and to zero behind it). Waves to different equilibria are shown to be related by Doob h-transforms. *Large-deviation theory* provides heuristic links between alternative descriptions of minimum wave speeds, rigorous algebraic proofs of which are provided.

The numerical work concentrates on the initial value problem. Finite difference methods are used to approximate the differential equation (which is most effective via use of the characteristics). The probabilistic system is simulated to obtain an alternative approximation to the solution to the initial value problem using the probabilistic representation.